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**Filling the Gap between Lower- $C^1$  and Lower- $C^2$  Functions**

The classes of lower- $C^{1,\alpha}$  functions ( $0 < \alpha \leq 1$ ), that is, functions locally representable as a maximum of a compactly parametrized family of continuously differentiable functions with  $\alpha$ -Hölder derivative, are hereby introduced. These classes form a strictly decreasing sequence from the larger class of lower- $C^1$  towards the smaller class of lower- $C^2$  functions, and can be analogously characterized via perturbed convex inequalities or via appropriate generalized monotonicity properties of their subdifferentials. Several examples are provided and a complete classification is given.

**Keywords:** Maximum function, lower- $C^{1,\alpha}$  function,  $\alpha$ -weakly convex function,  $\alpha$ -hypomonotone operator.

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