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Variational Analysis for a Class of Minimal Time Functions in Hilbert Spaces

This paper considers the parameterized infinite dimensional optimization problem

$$\text{minimize} \quad \big\{t \geq 0 : \ S \cap \{x + tF\} \neq \emptyset\big\},$$

where S is a nonempty closed subset of a Hilbert space H and $F \subseteq H$ is closed convex satisfying $0 \in \text{int } F$. The optimal value $T(x)$ depends on the parameter $x \in H$, and the (possibly empty) set $S \cap (x + T(x)F)$ of optimal solutions is the "F-projection" of x into S. We first compute proximal and Fréchet subgradients of $T(\cdot)$ in terms of normal vectors to level sets, and secondly, in terms of the F-projection. Sufficient conditions are also obtained for the differentiability and semiconvexity of $T(\cdot)$, results which extend the known case when F is the unit ball.