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# An Extension of the Serrin's Lower Semicontinuity Theorem

We present a new extension of a celebrated Serrin's lower semicontinuity theorem. We consider an integral of the calculus of variation $\int_\Omega f(x,u,Du)\,dx$ and we prove its lower semicontinuity in $W^{1,1}_{loc}(\Omega)$ with respect to the strong $L^1_{loc}$ norm topology, under the usual *continuity* and *convexity* property of the integrand $f(x,s,\xi)$, only assuming a mild (more precisely, *local*) condition on the independent variable $x \in \mathbb{R}^n$, say *local Lipschitz continuity*, which - we show with a specific counterexample - cannot be replaced, in general, by local *Hölder continuity*.

**Keywords**: Lower semicontinuity, strong convergence in L1, convex functions, local Lipschitz continuity, local Hoelder continuity, calculus of variations.

**MSC**: 49J45; 35D05