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On Subgradients of Spectral Functions

Let $F : \mathbf{S}(m) \to \overline{\mathbb{R}}$ be a *spectral function* (i.e. $\mathbf{S}(m)$ is the space of $m \times m$ real symmetric matrices, $\forall O \in \mathbf{O}(m), \forall X \in \mathbf{S}(m),\ F(OX{}^tO) = F(X)$, where $\mathbf{O}(m)$ is the orthogonal group and ${}^tO$ is the transpose of O). We associate to it the symmetric function $s_F : \mathbb{R}^m \to \overline{\mathbb{R}}$ by restricting it to the subspace of diagonal matrices. In this work, on the one hand, we give a new, natural proof of the formula which binds the Fréchet subgradients of a spectral function F and the Fréchet subgradients of the function s_F (identical formulas follow for the subgradients and the horizon subgradients); on the other hand we deduce from the previous results and from convexity arguments that, in the general case, a similar formula holds for the Clarke subgradients.

Keywords: Spectral function, eigenvalues, eigenvalue optimization, perturbation theory, Clarke subgradient, nonsmooth analysis.

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