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Separation Theorems for Abstract Convex Structures

Let S be a nonvoid set endowed with a "segment structure" which generalizes the notion of a segment in a linear space, and which allows to define affine functions. The problem is treated, whether a pair of sets $X, Y \subset S$ can be separated by some affine function $f : S \to \mathbb{R}$. Here separation means $f(y) \geq f(z)$, $y \in Y$, $z \in Z$ in its weakest and $\inf_{y \in Y} f(y) > \sup_{z \in Z} f(z)$ in its strongest form. Several solutions of this problem are presented as a consequence of von Neumann's minimax theorem. As special cases we obtain all the classical separation theorems for linear spaces, linear topological spaces, locally convex spaces, normed spaces, etc., but also new results for convex metric spaces are derived.

Keywords: Segment structure, affine function, separation of sets, convex metric space.

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