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A Residual Class of Holomorphic Functions

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Abstract. In their 1975 landmark paper, D. D. Bonar and F. W. Carroll have shown that, in the sense of category, there exists a residual class $\mathcal{SA}$ of "strongly annular" holomorphic functions in the open unit disk $\mathbb{D}$ such that, for each f in $\mathcal{SA}$, there exists an open subset $\mathcal{O}_{f,\infty}$ of $\mathbb{D}$ such that (1) $\mathcal{O}_{f,\infty}$ contains a sequence of concentric circles of increasing radii converging to the unit circle and (2) $f(z) \to \infty$ as $|z| \to 1$ through $\mathcal{O}_{f,\infty}$. Because circles have 2-dimensional Lebesgue measure zero, it has been an open question as to whether the set $\mathcal{O}_{f,\infty}$ could be chosen to have 2-dimensional measure-theoretic thickness. Here we give a definitive answer to that question. We show that for most functions f in $\mathcal{SA}$, the set $\mathcal{O}_{f,\infty}$ can be chosen so that it has upper global metric density 1. Even more, we show that for most functions f in $\mathcal{SA}$ and for every complex value ω there exists an open subset $\mathcal{O}_{f,\omega}$ of $\mathbb{D}$ that has upper global metric density 1 such that $f(z)$ converges to ω as $|z| \to 1$ through $\mathcal{O}_{f,\omega}$.

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