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Some Estimates for the Derivatives of Rational Functions

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Abstract. Let $\mathcal{P}_n$ be the class of all polynomials of degree at most n. It is known that if $f \in \mathcal{P}_n$ and $|f(z)| \leq 1$ on the unit circle, then $|f'(z)| \leq n|z|^{n-1}$ outside the unit disk. We present an 'extension' of this result to rational functions having all their poles in the open unit disk. Some inequalities involving $|f(z)|$, $|f'(z)|$ and $|f''(z)|$ are also proved in this paper. The last section contains an L^2 inequality for the derivative of a rational function.

Keywords. Polynomials, Bernstein's inequality, rational functions, Pick's inequality.

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