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An Extremal Problem for the Hyperbolic Metric on Denjoy Domains

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Abstract. Suppose that Ω is a domain in the extended complex plane and assume Ω contains the origin and that the boundary of Ω lies on the interval $[-1, 1]$ and has total length $2m$, $0 < m < 1$. We study the problem of finding the infimum of the density of the hyperbolic metric $\lambda(0, \Omega)$ at the origin among all such domains Ω . We show that if m is sufficiently large, the infimum is attained uniquely for the doubly connected domain which is symmetric in the imaginary axis. This result improves an estimate of A. Yu. Solynin. We also show that for sufficiently small m the above domain is not any more extremal.

Keywords. Hyperbolic metric, polarization, Denjoy domain, complete elliptic integral.

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