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On Uniform Approximation of Rational Perturbations of Cauchy Integrals

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Abstract. Let $[c,d]$ be an interval on the real line and μ be a measure of the form $d\mu = \dot{\mu} d\omega_{[c,d]}$ with $\dot{\mu} = h\hbar$, where $\hbar(t) = (t-c)^{\alpha_c}(d-t)^{\alpha_d}$, $\alpha_c, \alpha_d \in [0, 1/2)$, h is a Dini-continuous non-vanishing function on $[c,d]$ with an argument of bounded variation, and $\omega_{[c,d]}$ is the normalized arcsine distribution on $[c,d]$. Further, let p and q be two polynomials such that $\deg(p) < \deg(q)$ and $[c,d] \cap \mathrm{z}(q) = \emptyset$, where $\mathrm{z}(q)$ is the set of the zeros of q. We show that AAK-type meromorphic as well as diagonal multipoint Padé approximants to

$$\mathfrak{f}(z) := \int \frac{d\mu(t)}{z-t} + \left(\frac{p}{q}\right)(z)$$

converge locally uniformly to $\mathfrak{f}$ in $\mathfrak{D}_{\mathfrak{f}} \cap \mathbb{D}$ and $\mathfrak{D}_{\mathfrak{f}}$, respectively, where $\mathfrak{D}_{\mathfrak{f}}$ is the domain of analyticity of $\mathfrak{f}$ and $\mathbb{D}$ is the unit disk. In the case of Padé approximants we need to assume that the interpolation scheme is "nearly" conjugate-symmetric. A noteworthy feature of this case is that we also allow the density $\dot{\mu}$ to vanish on (c,d), although in a strictly controlled manner.

Keywords. Strong asymptotics, non-Hermitian orthogonality, meromorphic approximation, rational approximation, multipoint Padé approximation.

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