Ludwig Kuznia and Erik Lundberg

Fixed Points of Conjugated Blaschke Products with Applications to Gravitational Lensing

CMFT 9 No.2 (2009), 435–442. [ISSN 1617-9447]

Abstract. A conjecture in astronomy was recently resolved as an accidental corollary to a theorem regarding zeros of certain planar harmonic maps. As a step towards extending the Fundamental Theorem of Algebra, the theorem gave a bound of $5n-5$ for the number of zeros of a function of the form $r(z)-\bar{z}$, where $r(z)$ is rational of degree n. In this paper, we will investigate the case when $r(z)$ is a Blaschke product. The resulting (sharp) bound is $n+3$ and the proof is simple. We discuss an application to gravitational lenses consisting of collinear point masses.

Keywords. Blaschke product, gravitational lens, collinear point masses, proper self-maps.

2000 MSC. Primary 30D05; Secondary 83C99, 26C15.

Received. August 22, 2008, in revised form September 22, 2008.

Published online. December 18, 2008.