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**On Hyperbolic Divided Differences
and the Nevanlinna-Pick Problem**

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Abstract. Starting from the notion of the complex pseudo-hyperbolic distance and the hyperbolic difference quotient introduced by A. F. Beardon and D. Minda in [1], we define hyperbolic divided differences for unimodularly bounded holomorphic functions in the complex unit disk and investigate their mapping properties. In particular, we show that they operate on Blaschke products in the same way as the ordinary divided differences act on polynomials. As a simple corollary we obtain a multi-point Schwarz-Pick Lemma.

Using these concepts we investigate the classical interpolation problem of Pick and Nevanlinna and reformulate the Nevanlinna-Schur algorithm in terms of hyperbolic divided differences. This leads to a scheme that (formally) coincides with Newton's algorithm for polynomial interpolation.

Keywords. Hyperbolic geometry, divided differences, Nevanlinna-Pick interpolation, Newton interpolation, Schwarz Lemma.

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- 1 A. F. Beardon and D. Minda, A multi-point Schwarz-Pick lemma, *J. Anal. Math.* **92** (2004), 81–104.