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**A Riemann Mapping Theorem for
Two-Connected Domains in the Plane**

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Abstract. We show how to express a conformal map Φ of a general two-connected domain in the plane, such that neither boundary component is a point, onto a representative domain of the form $\mathcal{A}_r = \{z: |z + 1/z| < 2r\}$, where $r > 1$ is a constant. The domain \mathcal{A}_r has the virtue of having an explicit algebraic Bergman kernel function, and we shall explain why it is the best analogue of the unit disc in the two-connected setting. The map Φ will be given as a simple and explicit algebraic function of an Ahlfors map of the domain associated to a specially chosen point. It will follow that the conformal map Φ can be found by solving the same extremal problem that determines a Riemann map in the simply connected case. In the last section, we show how these results can be used to give formulae for the Bergman kernel in two-connected domains.

Keywords. Ahlfors map, Bergman kernel, Szegő kernel.

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