Janis Meyer

Cauchy Potentials with Angular Density Measures and a Generalisation of a Theorem of Keldysh

CMFT 9 No.1 (2009), 161–184. [ISSN 1617-9447]

Abstract. Let μ be a locally finite complex valued measure on $\mathbb{C}$ with an angular density with respect to a proximate order $\rho(r)$ and let

$$f_\mu(z) = z^p \iint_{\mathbb{C}} \frac{d\mu(\zeta)}{\zeta^p(z-\zeta)}$$

be the canonical Cauchy potential of μ. The main part of this paper is devoted to an estimate of $f_\mu(re^{i\phi})$ uniformly in ϕ generalising some results of A. Gol'dberg, N. Korenkov and N. V. Zabolotskii on the asymptotic expansion of the logarithmic derivative of entire functions. As application we give a partial answer to a problem raised by A. Eremenko, J. Langley and J. Rossi concerning the evaluation of the Nevanlinna deficiency of zeros of functions of the form

$$f(z) = \sum_{k=1}^{\infty} \frac{a_k}{z - z_k}, \qquad \sum_{|z_k|\neq 0}^{\infty} \left|\frac{a_k}{z_k}\right| < \infty$$

in terms of

$$\limsup_{r\to\infty} \frac{\log^+ \left|\sum_{|z_k|\leq r} a_k\right|}{\log r} = \rho.$$

We discuss the result from the point of view of a theorem of Keldysh.

Keywords. Cauchy potentials, angular densities of measures, regularly distributed sequences, Nevanlinna theory.

2000 MSC. 30D30, 30D35, 31A15.

Received. November 30, 2007, in revised form February 2, 2008.

Published online. April 30, 2008.