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**Generalized Elliptic Integrals**

CMFT 9 No.1 (2009), 75–109. [ISSN 1617-9447]

**Abstract.** Jacobi's elliptic integrals and elliptic functions arise naturally from the Schwarz-Christoffel conformal transformation of the upper half plane onto a rectangle. In this paper we study generalized elliptic integrals which arise from the analogous mapping of the upper half plane onto a quadrilateral and obtain sharp monotonicity and convexity properties for certain combinations of these integrals, thus generalizing analogous well-known results for classical conformal capacity and quasiconformal distortion functions. An algorithm for the computation of the modulus of the quadrilateral is given.

**Keywords.** Generalized elliptic integrals, modulus of a quadrilateral.

**2000 MSC.** Primary 33B15, 33C05; Secondary 30C62.

**Received.** March 17, 2007, in revised form February 7, 2008.

**Published online.** March 7, 2008.