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A Note on the Hayman-Wu Theorem

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Abstract. The Hayman-Wu Theorem states that the preimage of a line or circle L under a conformal mapping from the unit disc $\mathbb{D}$ to a simply-connected domain Ω has total Euclidean length bounded by an absolute constant. The best possible constant is known to lie in the interval $[\pi^2, 4\pi)$, thanks to work of Øyma and Rohde. Earlier, Brown Flinn showed that the total length is at most π^2 in the special case in which $L \subset \Omega$. Let r be the anti-Möbius map that fixes L pointwise. In this note we extend the sharp bound π^2 to the case where each connected component of $\Omega \cap r(\Omega)$ is bounded by one arc of $\partial\Omega$ and one arc of $r(\partial\Omega)$. We also strengthen the bounds slightly by replacing Euclidean length with the strictly larger spherical length on $\mathbb{D}$.

Keywords. Hyperbolic convexity, conformal reflection.

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