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How to Find a Measure from its Potential

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Abstract. We consider the problem of finding a measure from the given values of its logarithmic potential on the support. It is well known that a solution to this problem is given by the generalized Laplacian. The case of our main interest is when the support is contained in a rectifiable curve, and the measure is absolutely continuous with respect to the arclength on this curve. Then the generalized Laplacian is expressed by a sum of normal derivatives of the potential. Such a representation is already available for smooth curves, and we show it holds for any rectifiable curve in the plane. We also relax the assumptions imposed on the potential.

Finding a measure from its potential often leads to another closely related problem of solving a singular integral equation with Cauchy kernel. The theory of such equations is well developed for smooth curves. We generalize this theory to the class of Ahlfors regular curves and arcs, and characterize the bounded solutions on arcs.

Keywords. Potential, measure, boundary values, Cauchy singular integral, integral equations.

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