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Fourier-Bessel Series
for Second-Order and Fourth-Order
Bessel Differential Equations

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Abstract. In this paper we look at the Hilbert function space framework for Fourier-Bessel series, based on linear differential operators generated by the second-order Bessel differential equation and the fourth-order Bessel-type differential equation. In the second-order case attention is restricted to the differential equation for Bessel functions of order zero

$$-(xy'(x))' = \lambda xy(x) \qquad \text{for all } x \in (0,1],$$

where $\lambda \in \mathbb{C}$, the complex plane, is the spectral parameter. In the fourth-order case we concentrate on the Bessel-type differential equation

$$(xy''(x))'' - ((9x^{-1} + 8M^{-1}x)y'(x))' = \Lambda xy(x) \qquad \text{for all } x \in (0,1],$$

where $\Lambda \in \mathbb{C}$ is the spectral parameter, and $M > 0$ is a given parameter. In both cases the analysis is concerned with the theory of unbounded linear operators, generated by the differential equation, in the Hilbert function space $L^2((0,1);x)$. The analysis depends on new results in special function theory to develop properties of the solutions of the fourth-order Bessel-type differential equation, in particular the series expansions of these solutions at the regular singularity at the origin of $\mathbb{C}$.

Keywords. Fourier-Bessel series, Bessel functions, Bessel-type functions.

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