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Derivation-Invariant Subspaces of C^∞

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Abstract. Let $C^\infty(a,b)$ be the Fréchet space of all complex-valued infinitely differentiable functions on a (finite or infinite) interval $(a,b) \subset \mathbb{R}$. Let $L \subset C^\infty(a,b)$ be a closed subspace such that $DL \subset L$, where $D = \frac{d}{dx}$. Then the spectrum σ_L of D on L is either the whole complex plane, or a discrete possibly void set of eigenvalues λ, each one with some finite multiplicity $m_\lambda \in \mathbb{N}$ such that the monomial exponentials $e_{\lambda,j}(x) = x^j \exp(\lambda x)$, $0 \le j \le m_\lambda - 1$ belong to L. If the spectrum is void there is a relatively closed interval $I \subset (a,b)$ such that L consists of those functions from $C^\infty(a,b)$ which vanish identically on I. The interval may reduce to a point in which case L consists of the functions that vanish together with all their derivatives at that point.

Keywords. Differentiation operator, spectrum, nearly invariance, Fourier transform.

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