Daoud Bshouty and Abdallah Lyzzaik

Uniqueness of Harmonic Mappings into Strictly Starlike Domains

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Abstract. Let Ω be a bounded simply connected domain containing a point w_0 and whose boundary is locally connected, $\mathbb{D} = \{z\colon |z| < 1\}$ be the open unit disc, and $\omega : \mathbb{D} \to \mathbb{D}$ be an analytic function. It is known that the elliptic differential equation $\overline{f_z}/f_z = \omega$ admits a one-to-one solution normalized by $f(0) = w_0$, $f_z(0) > 0$, and maps $\mathbb{D}$ into Ω such that (i) the unrestricted limit $f^*(e^{it}) = \lim_{z\to e^{it}} f(z)$ exists and belongs to $\partial\Omega$ for all but a countable subset E of the unit circle $\mathbb{T} = \partial\mathbb{D}$, (ii) f^* is a continuous function on $\mathbb{T} \setminus E$ and for every $e^{is} \in E$ the one-sided limits $\lim_{t\to s^+} f^*(e^{it})$ and $\lim_{t\to s^-} f^*(e^{it})$ exist, belong to $\partial\Omega$, and are distinct, and (iii) the cluster set of f at $e^{is} \in E$ is the straight line segment joining the one-sided limits $\lim_{t\to s^+} f^*(e^{it})$ and $\lim_{t\to s^-} f^*(e^{it})$. In this paper it is shown that this solution is unique if Ω is a strictly starlike domain with respect to w_0 whose boundary is rectifiable.

Keywords. Harmonic mapping, analytic dilatation, strictly starlike domains, elliptic differential equation.

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