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Dieudonné Points of Holomorphic Self-Maps of Regions

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Abstract. Schwarz’s Lemma says that if a holomorphic self-map f of the unit disc fixes the origin then $|f'(0)| \leq 1$. In 1931, Dieudonné extended this by showing that $|f'(z)| \leq 1$ when $|z| < \sqrt{2} - 1$. We show that there is no analogue of Dieudonné’s Lemma for a simply connected hyperbolic region unless the region is a disc and the fixed point is the centre. We also discuss the analogue of Dieudonné’s result for multiply connected regions. In order to do this, we establish a local version of the Aumann-Carathéodory Rigidity Theorem, and we determine the maximal number of points fixed by some non-trivial conformal automorphism of a region in terms of its connectivity. More generally we investigate the topological relationship between the group of conformal automorphisms and its complement as subsets of the space of holomorphic self-maps of a hyperbolic region. For a simply connected hyperbolic region Ω we give sharp conformally invariant bounds on the hyperbolic distance between a holomorphic self-map f with a fixed point a and the hyperbolic rotation about a through angle $\arg f'(a)$.

Keywords. Hyperbolic regions, Euclidean contractions, fixed points.

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