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**On the Lower Order of Locally Univalent Functions**

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**Abstract.** Let  $f$  be analytic and  $f'(z) \neq 0$  in  $\mathbb{D}$  and let

$$A_f(z) = \frac{1 - |z|^2}{2} \frac{f''(z)}{f'(z)} - \bar{z} \quad \text{for } z \in \mathbb{D}.$$

Many properties of  $f$  can be described by the (linear-invariant) order

$$\sup_{z \in \mathbb{D}} |A_f(z)|.$$

The work of Avkhadiiev and Wirths led to the introduction of the lower order of  $f$  defined by  $\inf_{z \in \mathbb{D}} |A_f(z)|$ . It is perhaps a surprise that there are many (necessarily unbounded) functions of positive lower order. This paper studies some properties of these functions.

**Keywords.** Locally univalent, lower order, linear-invariant, Poincaré metric, trajectory.

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