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Some Relatives of the Hardy-Stein-Spencer Identities

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Abstract. Let f be a regular function, $r > 0$ and w a complex number. Denote by $n(w)$ the total number of roots of $f - w$ that lie in the disc $\{|z| < r\}$. An expression for the (two-dimensional) Fourier transform of n is derived from the Hardy-Stein-Spencer identities, and some consequences of it are explored. In particular, taking account of another identity due to Stein, we obtain a triple identity involving the real part of f , when this is positive, which doesn't appear to have been noticed before.

Keywords. Regular function, integral means, Hardy-Stein-Spencer identities, Fourier transform.

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