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**Majorization of the Modulus of Continuity
of Analytic Functions**

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Abstract. Let G be a bounded domain in the complex plane, let f be analytic in G and continuous in \overline{G} , and let μ be a majorant, that is, a non-negative non-decreasing function defined for $t \geq 0$ such that $\mu(2t) \leq 2\mu(t)$ for all $t \geq 0$. Suppose that $z_1 \in \partial G$ and that $|f(z_1) - f(z_2)| \leq \mu(|z_1 - z_2|)$ for all $z_2 \in \partial G$. We show that then $|f(z_1) - f(z_2)| \leq C\mu(|z_1 - z_2|)$ for all $z_2 \in G$ where $C = 3456$. If the assumption is made for all $z_1, z_2 \in \partial G$, then the conclusion holds for all $z_1, z_2 \in \overline{G}$. Earlier such a result, with an absolute constant C , had only been known when G is simply or doubly connected. The same result holds when G is an open set with only bounded components. We also give a survey of results on this type of problems, and explain the reductions that can be made.

Keywords. Analytic functions, modulus of continuity, majorization, maximum principle.

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