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The Structure of the Semigroup of Proper Holomorphic Mappings of a Planar Domain to the Unit Disc

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Abstract. Given a bounded n-connected domain Ω in the plane bounded by n non-intersecting Jordan curves and given one point b_j on each boundary curve, L. Bieberbach proved that there exists a proper holomorphic mapping f of Ω onto the unit disc that is an n-to-one branched covering with the properties: f extends continuously to the boundary and maps each boundary curve one-to-one onto the unit circle, and f maps each given point b_j on the boundary to the point 1 in the unit circle. We shall modify a proof by H. Grunsky of Bieberbach's result to show that there is a rational function of $2n+2$ complex variables that generates all of these maps. In fact, we show that there are two Ahlfors maps f_1 and f_2 associated with the domain such that any such mapping is given by a fixed linear fractional transformation mapping the right half plane to the unit disc composed with $c\,R + i\,C$, where R is a rational function of the $2n+2$ functions $f_1(z), f_2(z)$ and $f_1(b_1), f_2(b_1), \ldots, f_1(b_n), f_2(b_n)$, and where c and C are arbitrary real constants subject to the condition $c > 0$. We also show how to generate *all* the proper holomorphic mappings to the unit disc via the rational function R.

Keywords. Poisson kernel, Grunsky maps.

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