

L. E. Fraenkel

On the Increase of Capacity with Asymmetry

CMFT 8 No.1 (2008), 203–224. [ISSN 1617-9447]

Abstract. Let E be a compact, starlike set in \mathbb{R}^N , $N \geq 2$, that is very close to a ball B of the same area or volume. This paper presents inequalities, for logarithmic capacity if $N = 2$ or for capacity if $N \geq 3$, of the form

$$\text{lcap } E \geq \exp\{K_2\alpha(E)^2\} \text{lcap } \bar{B} \quad \text{or} \quad \text{cap } E \geq \{1 + K_N\alpha(E)^2\} \text{cap } \bar{B},$$

where $\alpha(E)$ is a modulus of asymmetry that measures the departure of the shape of E from that of B . The results are far less general than those of Hansen and Nadirashvili for $N = 2$ and those of Hall, Hayman and Weitsman for $N \geq 3$, but (for the particular sets considered) the present inequalities are somewhat sharper.

Keywords. Capacity, isoperimetric inequalities, potential theory.

2000 MSC. 31C15, 35J85.

Received. January 31, 2007.

Published online. July 25, 2007.