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Schwarzian Derivatives and Uniform Local Univalence

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Abstract. Quantitative estimates are obtained for the (finite) valence of functions analytic in the unit disk with Schwarzian derivative that is bounded or of slow growth. A harmonic mapping is shown to be uniformly locally univalent with respect to the hyperbolic metric if and only if it has finite Schwarzian norm, thus generalizing a result of B. Schwarz for analytic functions. A numerical bound is obtained for the Schwarzian norms of univalent harmonic mappings.

Keywords. Analytic function, valence, harmonic mapping, Schwarzian derivative, uniform local univalence, Schwarzian norm, minimal surface, harmonic lift.

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