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Starlike Functions in the Hornich Space

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Abstract. We will show that the set of starlike univalent functions in $\mathbb{D}$ is starlike in the Hornich space, i.e. for starlike functions f and $0 \leq \alpha \leq 1$ the function $\int_0^z (f'(\zeta))^\alpha \, d\zeta$ is also starlike. This solves a problem given by Kim, Ponnusamy and Sugawa in [6]. An important step in proving this result will be to show that for starlike functions f and $z \in \mathbb{D}$ we have $|\int_0^1 \arg(z/\gamma'(t)) \, dt| < \pi/2$, where $\gamma(t) := f^{-1}(tf(z))$, $0 \leq t \leq 1$.

Keywords. Starlike functions; Hornich operations; Integral transform.

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