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**Quaternions, Evaluation of Integrals  
and Boundary Value Problems**

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**Abstract.** In an attempt to enhance the accessibility of the beautiful theory of quaternions, we first revisit this theory emphasising that it provides the proper generalisation of the theory of complex analysis. In particular, we discuss the quaternionic generalisations of the following fundamental complex analytic notions: analytic functions, Cauchy’s Theorem, Cauchy’s integral formula, Taylor series, Laurent series, Residue Theorem, the Pompeiu (or Cauchy-Green, or Dbar) formula, and the Plemelj-Sokhotzki formulae. We then present two applications of the theory of quaternions, which provide generalisations of the analogous complex analytic applications: (a) the solution of certain boundary value problems for the Poisson equation in four dimensions; (b) the explicit computation of certain three dimensional real integrals.

**Keywords.** Quaternions, boundary value problems, complex analysis.

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