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**Boundary Behaviour of Functions  
in Weighted Dirichlet Spaces**

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**Abstract.** We consider the class  $\mathcal{D}_W$  of holomorphic functions  $f(z) = \sum a_n z^n$  in the unit disc for which  $\sum W(n)|a_n|^2 < \infty$ , where the weight function  $W$  satisfies standard regularity conditions. We show that if  $\sum 1/(nW(n)) < \infty$  and  $f \in \mathcal{D}_W$ , then the radial variation  $L_f(\theta) = \int_0^1 |f'(re^{i\theta})| dr$  is finite outside an exceptional set of capacity zero, where the kernel associated with the capacity depends on  $W$ . It is known that if  $\sum 1/(nW(n)) = \infty$ , then there exist functions in  $\mathcal{D}_W$  with  $L_f(\theta) = \infty$  for every  $\theta$ . We also show that it is a consequence of known results that if  $f \in \mathcal{D}_W$  and  $\sum 1/W(n) = \infty$ , then  $f$  has finite radial, and non-tangential, limits outside certain exceptional sets.

**Keywords.** Weighted Dirichlet spaces, radial variation, radial limits, exceptional sets, zero capacity.

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