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Sharp Bounds for Sums of Coefficients of Inverses of Convex Functions

CMFT 7 No.1 (2007), 105–109. [ISSN 1617-9447]

Abstract. Let $\mathbb{D}$ denote the open unit disc and $f\colon \mathbb{D} \to \mathbb{C}$ be holomorphic and injective in D such that $f(D)$ is a convex domain and $f(0) = f'(0) - 1 = 0$. Let F be the inverse function of f defined in a neighbourhood of the origin and $k \in \mathbb{N}$. We consider the Taylor expansions

$$(F(w))^k = \sum_{n=k}^{\infty} A_{n,k} w^n.$$

We prove that the inequality

$$\left| \sum_{k=1}^{n} A_{n,k} \right| \leq 2^{n-1}$$

is valid for any $n \in \mathbb{N}$ and that equality occurs in this inequality for a fixed $n \geq 2$ if and only if $f(z) = z/(1+z)$.

Keywords. Taylor coefficients, convex functions, inverse functions, bounded functions.

2000 MSC. 30C50, 30C45, 30D50.

Received. July 4, 2005, in revised form July 20, 2006.