Peter A. Clarkson

Special Polynomials Associated with Rational Solutions of the Painlevé Equations and Applications to Soliton Equations

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Abstract. Rational solutions of the second, third and fourth Painlevé equations can be expressed in terms of special polynomials defined through second order bilinear differential-difference equations which are equivalent to the Toda equation. In this paper the structure of the roots of these special polynomials, as well as the special polynomials associated with algebraic solutions of the third and fifth Painlevé equations and equations in the $\mathrm{P_{II}}$ hierarchy, are studied. It is shown that the roots of these polynomials have an intriguing, highly symmetric and regular structure in the complex plane. Further, using the Hamiltonian theory for the Painlevé equations, other properties of these special polynomials are studied. Soliton equations, which are solvable by the inverse scattering method, are known to have symmetry reductions which reduce them to Painlevé equations. Using this relationship, rational solutions of the Korteweg-de Vries and modified Korteweg-de Vries equations and rational and rational-oscillatory solutions of the non-linear Schrödinger equation are expressed in terms of these special polynomials.

Keywords. Hamiltonians, Painlevé equations, rational solutions.

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