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The Isoperimetric Inequality via Approximation Theory and Free Boundary Problems

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Abstract. In this survey paper, we examine the isoperimetric inequality from an analytic point of view. We use as a point of departure the concept of *analytic content* in approximation theory: this approach reveals ties to overdetermined boundary problems and hydrodynamics. In particular, we look at problems connected to determining the shape of an electrified droplet or equivalently, that of an air bubble in fluid flow. We also discuss the connection with the Schwarz function and quadrature domains. Finally, we survey some known generalizations to higher dimensions and list many open problems that remain. This paper is an expanded version of the plenary talk given by the second author at the fifth CMFT Conference in Joensuu, Finland, in June 2005.

Keywords. Analytic content, free boundary problems, hydrodynamics, isoperimetric inequality, quadrature domains, rational approximation.

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