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Euclidean Properties of Hyperbolic Polar Coordinates

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Abstract. In a simply connected hyperbolic region hyperbolic polar coordinates possess global Euclidean properties similar to those of hyperbolic polar coordinates about the origin in the unit disk if and only if the region is convex. For example, the Euclidean distance between travelers moving at unit hyperbolic speed along distinct hyperbolic geodesic rays emanating from a common point is increasing if and only if the region is convex. A consequence of this is that the ends of distinct hyperbolic geodesic rays in a convex region cannot be too close. Uniform local versions of these Euclidean properties of hyperbolic polar coordinates hold if and only if the hyperbolic region is uniformly perfect.

Keywords. Hyperbolic metric, convex univalent functions, convex regions, starlike regions.

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