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Convergence of Rational Interpolants to Analytic Functions with Restricted Growth

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Abstract. Let D be a regular domain in $\overline{\mathbb{C}}$ with bounded boundary. For each $n \geq 1$, take points $A_n = \{a_{ni}\}_{i=0}^n$ in D and points $B_n = \{b_{ni}\}_{i=1}^n$ in $\overline{\mathbb{C}}$, such that $\bigcup B_n$ has no limit points in D . Let α_n and β_n be the normalized point counting measures of A_n and B_n respectively and α'_n and β'_n their swept measures onto ∂D . Denote by U_μ the logarithmic potential of the measure μ . It has been shown by Ambroladze and Wallin, that if for every weak-star limit point α of $\{\alpha_n\}$, we have $\alpha(D) > 0$ and also

$$\lim_{n \rightarrow \infty} \left[\sup_{z \in \partial D} \left(\frac{n+1}{n} U_{\alpha'_n}(z) - U_{\beta'_n}(z) \right) \right] = 0,$$

then for every bounded analytic function f on D , the rational interpolants r_n of degree n with poles at B_n interpolating to f at A_n , converge locally uniformly with geometric degree of convergence to f on D . We show that under a slightly stricter condition on $\{B_n\}$, the boundedness requirement on f can be replaced with the weaker growth condition

$$|f(z)| \leq Ad(z, \partial D)^q,$$

where $A > 0$, $q < 0$ and that this growth condition is also sharp in the sense described in this paper.

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