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**Conjugate Harmonic Functions in Euclidean Space:
a Spherical Approach**

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Abstract. Given a harmonic function U in a domain Ω in Euclidean space, the problem of finding a harmonic conjugate V , generalizing the well-known case of the complex plane, was considered in [4] in the framework of Clifford analysis. By the nature of the given construction, which is genuinely cartesian, this approach lead to geometric constraints on the domain Ω . In this paper we consider the problem in a larger class of domains, by a spherical approach. Starting from a real-valued function u , and singling out the radial direction, we explicitly construct a harmonic function of the form $w = e_r v$, with $v \in \text{span}(e_{\theta_1}, \dots, e_{\theta_{m-1}})$, such that $u+w$ is monogenic, i.e. a null solution of the Dirac operator. As an illustration, the construction is applied to important classes of homogeneous monogenic polynomials and functions. Finally, it is investigated to which extent the approach also applies to the complex plane case.

Keywords. Conjugate harmonic functions, Clifford analysis.

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