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Zeros of Hypergeometric Functions and the Norm of a Composition Operator

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Abstract. Let φ be an analytic self-map of the unit disk; let C_φ denote the corresponding composition operator acting on the Hardy space H^2 . Although the precise value of $\|C_\varphi\|$ is quite difficult to calculate, some progress has been made in the case when φ is a linear fractional map. A recent paper by Basor and Retsek demonstrates a connection between the norm of such an operator and the zeros of a particular hypergeometric series. Here we will pursue this line of inquiry further. We shall appeal to several results relating to hypergeometric series — many of which are quite old — to deduce more information about the norm of a composition operator, in particular about the spectrum of $C_\varphi^*C_\varphi$. Furthermore, we will use our knowledge of composition operators to establish an apparently new result pertaining to the zeros of hypergeometric series.

Keywords. Composition operator, operator norm, hypergeometric series.

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