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**Hardy's Generalization of e^z
and Related Analogs of Cosine and Sine**

CMFT 6 No.1 (2006), 1–14. [ISSN 1617-9447]

Abstract. In 1904, Hardy introduced an entire function depending on two parameters being a generalization of e^z . He had studied in detail its asymptotic properties and that of its zeros. We consider the two following *non-asymptotic* problems related to the zeros. (i) Determine values of the parameters such that all the zeros belong to the open left half-plane. For these values, the analogs of sine and cosine generated by Hardy's function have real, simple and interlacing zeros. (ii) Determine the number of real zeros as a function of the parameters.

Keywords. Class \mathcal{P} , integral representation, Levin's generalization of the Hermite-Biehler Theorem, logarithmic derivative, Rolle's Theorem.

2000 MSC. 30D10, 30D15.

Received. December 12, 2005.