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On Modified Biquaternionic Analysis in $\mathbb{C}^3$

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Abstract. Identify $\mathbb{C}^3$ with the set of reduced biquaternions of the form $z = z_0 + z_1 e_1 + z_2 e_2$, where $z_i \in \mathbb{C}$. For $\Omega \subset \mathbb{C}^3$, let $w\colon \Omega \to \mathbb{C}^3$ be a function $w = w_0 + w_1 e_1 + w_2 e_2$. We consider the solutions of

$$\begin{cases} z_2 \left(\dfrac{\partial w_0}{\partial z_0} - \dfrac{\partial w_1}{\partial z_1} - \dfrac{\partial w_2}{\partial z_2} \right) + k w_2 = 0, & k \in \mathbb{R} \\ \dfrac{\partial w_1}{\partial \bar{z}_2} = \dfrac{\partial w_2}{\partial \bar{z}_1}, & \\ \dfrac{\partial w_0}{\partial \bar{z}_1} = -\dfrac{\partial w_1}{\partial \bar{z}_0}, & \\ \dfrac{\partial w_0}{\partial \bar{z}_2} = -\dfrac{\partial w_2}{\partial \bar{z}_0}. & \end{cases}$$

Our system is a modification of a system introduced by Li Yucheng and Qiao Yuying and a complexification of a system introduced by H. Leutwiler. These M_k-solutions are connected with k-hyperbolic harmonic functions h by

$$w = \frac{\partial h}{\partial \bar{z}_0} - \frac{\partial h}{\partial \bar{z}_1} e_1 - \frac{\partial h}{\partial \bar{z}_2} e_2,$$

where h satisfies

$$z_2 \Delta h - 4k \frac{\partial h}{\partial \bar{z}_2} = 0.$$

The k-hyperbolic harmonic functions are also connected to polyharmonic ones, $\Delta^k h$ is harmonic.

We find basic properties for M_k-solutions and k-hyperbolic harmonic functions and examine their bases.

Keywords. Modified quaternionic analysis, biquaternions, hyperbolic harmonic functions.

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