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**On Keogh's Length Estimate
for Bounded Starlike Functions**

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Abstract. For a bounded starlike function f on the unit disc, we consider $L(r)$, the length of the image of the circle $|z| = r$. Keogh showed that $L(r) = \mathcal{O}(\log 1/(1-r))$ as $r \rightarrow 1$ and Hayman showed that this is the correct asymptotic. We give an alternative *geometric* construction which strengthens Hayman's result, showing that the constant in Keogh's original inequality is sharp. The analysis uses standard estimates on the hyperbolic metric of plane domains. The self-similarity of the construction allows for the examples to be expressed analytically. For context, we give a brief survey of related estimates on integral means and coefficients of univalent functions.

Keywords. Starlike, conformal mappings, self similar, integral means.

2000 MSC. Primary 30C45; Secondary 30C35.

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