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Zero Distributions for Polynomials Orthogonal with Weights over Certain Planar Regions

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Abstract. Let G be a bounded Jordan domain in $\mathbb{C}$ and let $w \not\equiv 0$ be an analytic function on G such that $\int_G |w|^2 dm < \infty$, where dm is the area measure. We investigate the zero distribution of the sequence of polynomials that are orthogonal on G with respect to $|w|^2 dm$. We find that such a distribution depends on the location of the singularities of the reproducing kernel $K_w(z,\zeta)$ of the space $\mathcal{L}^2_w(G) := \left\{ f \text{ analytic on } G : \int_G |f|^2 |w|^2 \, dm < \infty \right\}$. A fundamental theorem is given for the case when $K_w(\cdot,\zeta)$ has a singularity on ∂G for at least some $\zeta \in G$. To investigate the opposite case, we consider two examples in detail: first when G is the unit disk and w is meromorphic, and second when G is a lens-shaped domain and w is entire. Our analysis can also be applied for $w \equiv 1$ in the case when G is a rectangle or a special triangle. We also provide formulas for $K_w(\cdot,\zeta)$ that are of help for the determination of its singularities.

Keywords. Orthogonal polynomials, zeros of polynomials, kernel function, logarithmic potential, equilibrium measure.

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