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**On Schwarz's Lemma in Multiply Connected Domains**

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**Abstract.** Let  $G$  be a  $n$ -connected domain in the complex plane and let  $z_0 \in G$  be fixed. We consider the class  $H_1(G, z_0)$  of all holomorphic functions  $f$  of  $G$  into the unit disk  $E$  with  $f(z_0) = 0$ . For  $z \in G \setminus \{z_0\}$  the Carathéodory distance  $c(z, z_0)$  is defined as the maximum of  $|f(z)|$ ,  $f \in H_1(G, z_0)$ . We determine explicitly the balls  $\{z \in G : c(z, z_0) < r\}$  in the case  $n = 2$ . Grunsky proved that the extremal functions with respect to the Carathéodory distance are proper mappings of degree  $n$ . Here we add that only in the case  $n = 2$  each proper mapping in  $H_1(G, z_0)$  is somewhere in  $G$  extremal.

**Keywords.** Carathéodory distance, ring domains, extremal functions.

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