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Remarks on Complex Difference Equations

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Abstract. Halburd and Korhonen have shown that the existence of sufficiently many meromorphic solutions of finite order is enough to single out a discrete form of the second Painlevé equation from a more general class $f(z+1) + f(z-1) = R(z, f)$ of complex difference equations. A key lemma in their reasoning is to show that $f(z)$ has to be of infinite order, provided that $\deg_f R(z, f) \leq 2$ and that a certain growth condition for the counting function of distinct poles of $f(z)$ holds. In this paper, we prove a generalization of this lemma to higher order difference equations of more general type. We also consider related complex functional equations.

Keywords. Complex difference equation, complex functional equation, Nevanlinna theory, value distribution theory.

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