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Extremal Point Methods for Robin Capacity

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Abstract. The Robin capacity $\delta(A)$ of a compact, non-empty set $A \subset \partial\Omega$ with respect to a domain $\Omega \subset \widehat{\mathbb{C}}$ containing ∞ is defined by

$$\delta(A) = \delta(A, \Omega) = \exp\Big(\lim_{z\to\infty} -R(z) + \log|z|\Big),$$

where $R(z) = R(z, \infty)$ is the fundamental solution of a mixed boundary value problem with pole at ∞, where Dirichlet conditions are imposed on A and Neumann conditions on $\partial\Omega \setminus A$. P. Duren and M. Schiffer have discovered that it coincides with the minimal logarithmic capacity of $f(A)$ over all conformal mappings f of Ω with $f(z) = z + \mathcal{O}(1)$, $z \to \infty$.

In this article, effective methods for the numerical determination of $\delta(A)$ are developed. For this purpose the conformal invariant $\delta(A)/\operatorname{cap}(\partial\Omega)$ is related to other moduli of the given configuration like harmonic measure or conformal modulus. Then an effective extremal point discretization for these moduli based on Menke points is derived. If Ω is analytically bounded, the discretizations presented provide geometrically fast converging approximations to the considered moduli and thus to Robin capacity.

Keywords. Robin capacity, conformal invariants, extremal points.

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