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Extending a Theorem of Bergweiler and Langley Concerning Non-Vanishing Derivatives

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Abstract. We consider the differential operator Λ_k defined by

$$\Lambda_k(y) = \Psi_k(y) + a_{k-1}\Psi_{k-1}(y) + \cdots + a_1\Psi_1(y) + a_0,$$

where $a_0, \ldots, a_{k-1}$ are analytic functions of restricted growth and $\Psi_k(y)$ is a differential operator defined by $\Psi_1(y) = y$ and $\Psi_{k+1}(y) = y\Psi_k(y) + (\Psi_k(y))'$ for $k \in \mathbb{N}$. We suppose that $k \geq 3$, that F is a meromorphic function on an annulus $\mathcal{A}(r_0)$, and that $\Lambda_k(F)$ has all its zeros on a set E such that E has no limit point in $\mathcal{A}(r_0)$. We suppose also that all simple poles a of F in $\mathcal{A}(r_0) \setminus E$ have $\operatorname{res}(F, a) \notin \{1, \ldots, k-1\}$. We then deduce that F is a function of restricted growth in the Nevanlinna sense. This extends a theorem of Bergweiler and Langley [1]. We show also that this result does not hold when $a_0, \ldots, a_{k-1}$ are meromorphic functions.

Keywords. Meromorphic functions, Nevanlinna theory.

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1 W. Bergweiler and J. K. Langley, Non-vanishing derivatives and normal families, *J. Anal. Math.* **91** (2003), 353–367.