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Badly Approximable Unimodular Functions in Weighted L^p Spaces

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Abstract. A function u on the unit circle $\mathbb{T}$ is said to be badly approximable in the weighted space $L^p(\mathbb{T}, w)$ if $\|u + f\|_{L^p(\mathbb{T},w)} \geq \|u\|_{L^p(\mathbb{T},w)}$ for all $f \in H^\infty$. We prove that if an unimodular function u is badly approximable in $L^p(\mathbb{T}, w)$ for all $p \in (0, +\infty)$ and some non-zero weight w, then $\overline{u}$ is an inner function. We describe the inner functions Θ and the weights w on the unit circle $\mathbb{T}$ such that $\overline{\Theta}$ is badly approximable in $L^p(\mathbb{T}, w)$ for all $p > 0$. It turns out that, for given inner functions Θ, the class of all weights satisfying the above-mentioned condition depends only on the zero set of Θ. In other words, $\overline{\Theta}$ is badly approximable in $L^p(\mathbb{T}, w)$ for all $p \in (0, +\infty)$ if and only if $\overline{B}$ is badly approximable in $L^p(\mathbb{T}, w)$ for all $p \in (0, +\infty)$, where B is a Blaschke product with simple zeros and such that $\Theta^{-1}(0) = B^{-1}(0)$.

Keywords. Hardy spaces, inner functions, best approximation.

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