

Line Baribeau, Dominique Brunet, Thomas Ransford, and Jérémie Rostand
Iterated Function Systems, Capacity
and Green's Functions

CMFT 4 No.1 (2004), 47–58. [ISSN 1617-9447]

Abstract. Let $f_1, \dots, f_m: \mathbb{C} \rightarrow \mathbb{C}$ be maps satisfying

$$a_j|z - w| \leq |f_j(z) - f_j(w)| \leq b_j|z - w|, \quad z, w \in \mathbb{C}, j = 1, \dots, m,$$

where $0 < a_j \leq b_j < 1$, $j = 1, \dots, m$. Let E be the attractor of this iterated function system, namely the unique compact subset of \mathbb{C} satisfying $E = \bigcup_1^m f_j(E)$. Assume that E does not reduce to a singleton (i.e. that the maps f_j have no common fixed point).

We give a lower bound for the logarithmic capacity $c(E)$ of E in terms of the diameter $\text{diam}(E)$ and the constants $a_1, \dots, a_m, b_1, \dots, b_m$. We further prove that

$$c(E \cap \overline{D}(w, r)) \geq Cr^\alpha, \quad w \in E, 0 < r \leq \text{diam}(E),$$

where $C > 0$ and $\alpha = \max_j(\log a_j / \log b_j)$, and deduce that E is non-thin at every point of itself. Finally, in the case where $a_j = b_j$ for each j (so all the f_j are similarities), we give a simple proof that the Green's function of E is Hölder continuous, and obtain estimates for the exponent of Hölder continuity.

Keywords. Iterated function system, attractor, capacity, Green's function, Hölder continuous.

2000 MSC. Primary 31A15, Secondary 28A80.

Received. January 13, 2004, in revised form March 22, 2004.