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Conformality of the Apollonian Metric

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Abstract. The Apollonian metric a_D of a domain $D \subset \overline{\mathbb{R}^n}$ is rarely conformal. In fact, if it is conformal at one point then D is, up to a Möbius transformation, a complement of a convex body of constant width and if it is conformal at two points then D is a ball. We consider a quantity that measures the deviation of a_D from being conformal. This quantity is essential in comparing the Apollonian metric to hyperbolic and quasihyperbolic metrics. We show that this quantity is invariant under Möbius transformations and compute it for some standard domains. We then use it to obtain sharp estimates between any two of the Apollonian, hyperbolic and quasihyperbolic metrics on such domains.

Keywords. Apollonian metric, hyperbolic metric, quasihyperbolic metric, Möbius maps.

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