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Generalized Iteration

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Abstract. The main purpose of the Fatou-Julia theory is to study the global behaviour of the sequence (f^n) of iterates of a rational function f . In this survey article we consider generalized iteration which means that the iterated function f may vary from step to step. More precisely, let (f_n) be a sequence of rational functions, and let $F_n := f_n \circ \cdots \circ f_1$ be the sequence of forward compositions, and let the Fatou set and Julia set of (F_n) be defined as usual. Then, in general, most of the results of the Fatou-Julia theory fail to hold. On the other hand, under appropriate restrictions on the sequence (f_n) many results can be carried over to this more general situation, but the proofs are often completely different.

We also consider compositions of holomorphic self-maps f_n of the unit disk. In this case there is no need to deal with Fatou and Julia sets, and the main interest lies in the dynamics of (F_n) . It also makes sense to consider the sequence of backward compositions $\Phi_n := f_1 \circ \cdots \circ f_n$, because such sequences arise, for example, in continued fraction expansions.

Keywords. Iteration, random iteration, forward composition, backward composition, Fatou set, Julia set, dynamics.

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