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Punishing Factors for Angles

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Abstract. Let Ω and Π be two simply connected domains in the complex plane $\mathbb{C}$ which are not equal to the whole plane $\mathbb{C}$. We consider functions $f\colon \Omega \to \Pi$ analytic in Ω and we get estimates for $|f^{(n)}(z)|$, $z \in \Omega$, which are sharp in the following sense. Let $\lambda_\Omega(z)$ and $\lambda_\Pi(w)$ denote the reciprocal of the conformal radius of Ω in z and of Π in w, respectively. Inequalities of the type

$$\frac{|f^{(n)}(z)|}{n!} \le M_n(z,\Omega,\Pi)\frac{(\lambda_\Omega(z))^n}{\lambda_\Pi(f(z))}, \qquad z \in \Omega,$$

are considered where $M_n(z,\Omega,\Pi)$ does not depend on f and represents the smallest value possible at this place. We especially consider cases where Ω or Π is an angular domain H_α with opening angle $\alpha\pi$, $1 \le \alpha \le 2$. We determine $M_n(z,\Delta,H_\alpha)$ where Δ denotes the unit disk.

Furthermore, we prove identities and inequalities for

$$C_n(\Omega,\Pi) := \sup\{M_n(z,\Omega,\Pi) \mid z \in \Omega\}$$

for several cases where H_α plays the role of Ω or Π.

Keywords. Angular domains, derivatives of arbitrary order, conformal radius.

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