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**Airy Solutions of Painlevé's Second Equation**

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**Abstract.** We prove that any transcendental solution of Painlevé's second equation  $w'' = \alpha + zw + 2w^3$ , which has the form  $w = R(z, u)$ , with  $R$  rational in both variables and non-linear with respect to  $u$ , is obtained by repeated application of the Bäcklund transformation to some solution of the Riccati equation  $U' = \pm(z/2 + U^2)$ . In particular,  $\alpha = n + 1/2$ ,  $n \in \mathbb{Z}$ , and  $w$  has order of growth  $\varrho = 3/2$ . Moreover it is shown that  $u$  satisfies some Riccati differential equation  $u' = a(z) + b(z)u + c(z)u^2$  with rational coefficients.

**Keywords.** Painlevé, Airy and Riccati differential equation, Bäcklund transformation.

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